SUPPLEMENTARY SHEET 5
NEGATIVE EXPONENTS & SCIENTIFIC NOTATION
ELEMENTARY ALGEBRA - MAT 0024

Professor Howard Sorkin

The basic concept of the NEGATIVE EXPONENT is to take the INVERSE or RECIPROCAL of the base to the POSITIVE power.

RULE 1: $a^{-n} = \frac{1}{a^n}$

Proof: $$\frac{a^3}{a^5} = \frac{\overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot a \cdot a} = \frac{1}{a^2}$$

By DIVISION RULE: $$\frac{a^3}{a^5} = a^{3-5} = a^{-2}$$

Therefore: → $$a^{-2} = \frac{1}{a^2}$$

Examples:

a) $$3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

b) $$-2^{-2} = -\frac{1}{2^2} = -\frac{1}{4}$$

c) $$(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$$

d) $$-(-2)^{-3} = -\frac{1}{(-2)^3} = -\frac{1}{(-8)} = \frac{1}{8}$$

ORDER OF OPERATIONS: FIRST the EXPONENT then the DOUBLE NEGATIVE.

e) $2^{-1} + 5^{-1} = \frac{1}{2} + \frac{1}{5}$

FIRST write *without* the negative exponent, THEN do the addition problem.

Old way by finding least common denominators:

$$\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

New way to add/subtract two fractions that do not have a common factor other than 1 in their denominators:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

We know how to add 5 + 2 in our heads. We really do not have to write out this step!!

$$\frac{1}{2} + \frac{1}{5} = \left(\frac{5 + 2}{10}\right) = \frac{7}{10}$$

RULE 2: $\dfrac{1}{a^{-n}} = a^{n}$

Proof: $\dfrac{1}{a^{-3}} = \dfrac{1}{\frac{1}{a^{3}}} = 1 \div \frac{1}{a^{3}} = 1 \cdot a^{3} = a^{3}$

From RULE 1

THIS IS REALLY A <u>DIVISION</u> PROBLEM

Examples:

a) $\dfrac{1}{3^{-4}} = 3^{4} = 81$

b) $\dfrac{1}{(-5)^{-2}} = (-5)^{2} = 25$

c) $-\dfrac{1}{(-2)^{-3}} = -(-2)^{3} \; -(-8) = 8$

ORDER OF OPERATIONS: <u>FIRST</u> the EXPONENT then the DOUBLE NEGATIVE.

d) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$

Remember: A negative exponent means the **INVERSE** or **RECIPROCAL** of the **BASE** to the **POSITIVE** power

In this example $\frac{2}{3}$ is the **BASE**.

What we are doing is taking the **INVERSE** or **RECIPROCAL** of the **BASE** to the **POSITIVE** power.

MULTIPLICATION & DIVISION - SCIENTIFIC NOTATION

You learned about SCIENTIFIC NOTATION in Section 8.2. Suppose we want to find the answer to the following multiplication problem, written in STANDARD FORM.

$$(3.4 \times 10^4) \times (2 \times 10^3)$$

Since we know multiplication is COMMUTATIVE, we can rewrite this problem as follows:

$$3.4 \times 2 \times 10^4 \times 10^3 \;=\; 6.8 \times 10^7 \;=\; 68{,}000{,}000$$

Of course, there really is no need to rewrite this problem if we are able to calculate 3.4×2 and $10^4 \times 10^3$ in our heads.

We will now find the answer, in STANDARD FORM, to the following division problem:

$$\frac{2.4 \times 10^{-3}}{6 \times 10^2}$$

Since we want our answer to be in STANDARD FORM, when we DIVIDE we ALWAYS bring the 10^x which appears in the denominator (x is the power of the 10 in the denominator) UP and work out the answer. REMEMBER, when we move a number either UP or DOWN, the sign of the exponent changes (see RULE 1 and RULE 2 above).

Here are the steps involved with finding the answer to the DIVISION problem above.

1. Simplify the $\frac{2.4}{6}$ part of the problem.
2. Bring 10^2 UP (it now becomes 10^{-2}) and find this answer.
3. Write the final answer in STANDARD FORM.

$$\frac{2.4 \times 10^{-3}}{6 \times 10^2} = \frac{\overset{.4}{\cancel{2.4}} \times 10^{-3} \times 10^{-2}}{\cancel{6}} = .4 \text{ x } 10^{-5} = .000004$$

Bring 10^2 UP

These problems will test your understanding of negative and zero exponents as well as Scientific Notation. Simplify each and write all answers with only *POSITIVE* exponents.

1. 4^{-2}
2. $(-4)^{-2}$
3. -4^{-2}
4. $-(-4)^{-2}$
5. 5^{-3}
6. $(-5)^{-3}$
7. -5^{-3}
8. $-(-5)^{-3}$
9. $\left(\frac{1}{3}\right)^{-1}$
10. $\left(-\frac{1}{3}\right)^{-1}$
11. $-\left(\frac{1}{3}\right)^{-1}$
12. $-\left(-\frac{1}{3}\right)^{-1}$
13. $\left(\frac{2}{5}\right)^{-2}$
14. $\left(-\frac{2}{5}\right)^{-2}$
15. $-\left(\frac{2}{5}\right)^{-2}$
16. $-\left(-\frac{2}{5}\right)^{-2}$
17. 7^0
18 -7^0
19. $(-7)^0$
20. $2x^0$
21. $(2x)^0$
22. $-2x^0$
23. $(-2x)^0$
24. $3x^2yz^0$
25. $-4xyz^0$
26. $-4x(yz)^0$
27. $-4(x^2y^3z)^0$
28. $-3^0+(-3)^0$
29. $-3^0-(-3)^0$
30. $-3^0 + 3^0$
31. $-3^0 - 3^0$
32. $-4^{-1} - 3^{-1}$
33. $2\cdot 4^{-1} - 3^{-1}$
34. $2\cdot 4^{-1} - 4\cdot 3^{-1}$
35. $4^{-1} - 2\cdot 3^{-1}$
36. $2^{-2} + 2^{-1}$
37. $3\cdot 2^{-2} + 2^{-1}$
38. $2^{-1} - 3\cdot 2^{-2}$
39. $-2^{-2} - 2$
40. $1 - 3^{-4}$
41. $\left(\frac{4b^3}{12b^5}\right)^{-2}$
42. $\left(\frac{3x^4}{12x^2}\right)^{-3}$
43. $\left(\frac{15m^{-2}}{20m^{-5}}\right)^{-2}$
44. $\left(\frac{8n^{-5}}{12n^{-2}}\right)^{-3}$
45. $\frac{(x^5\, y^{-3})^2}{x^{-4}\, y^5}$
46. $\frac{(a^{-5}\, b^4)^3}{a^{-5}\, b^2}$
47. $\frac{(p^{-5}\, q^2)^{-2}}{p^{-5}\, q}$
48. $\frac{(m^4\, n^{-3})^{-3}}{m^{-2}\, n^{10}}$
49. $(7.3\times 10^5)\times(8\times 10^2)$
50. $(3\times 10^{-5})\times(2.6\times 10^3)$
51. $(5.32\times 10^3)\times(3\times 10^{-4})$
52. $(4.2\times 10^{-4})\times(4\times 10^{-1})$
53. $\frac{7.8\times 10^5}{3\times 10^3}$
54. $\frac{5.4\times 10^{-3}}{2\times 10^2}$
55. $\frac{3.4\times 10^4}{4\times 10^{-2}}$
56. $\frac{5.5\times 10^{-5}}{11\times 10^{-2}}$

☺☺☺☺☺☺☺☺☺☺☺☺☺☺

ANSWERS

1. $\frac{1}{16}$
2. $\frac{1}{16}$
3. $-\frac{1}{16}$
4. $-\frac{1}{16}$
5. $\frac{1}{125}$
6. $-\frac{1}{125}$
7. $-\frac{1}{125}$
8. $\frac{1}{125}$
9. 3
10. -3
11. -3
12. 3
13. $\frac{25}{4}$
14. $\frac{25}{4}$
15. $-\frac{25}{4}$
16. $-\frac{25}{4}$
17. 1
18. -1
19. 1
20. 2
21. 1
22. -2
23. 1
24. $3x^2y$
25. $-4xy$
26. $-4x$
27. -4
28. 0
29. -2
30. 0
31. -2
32. $-\frac{7}{12}$
33. $\frac{1}{6}$
34. $-\frac{5}{6}$
35. $-\frac{5}{12}$
36. $\frac{3}{4}$
37. $\frac{5}{4}$
38. $-\frac{1}{4}$
39. $-\frac{9}{4}$
40. $\frac{80}{81}$
41. $9b^4$
42. $\frac{64}{x^6}$
43. $\frac{16}{9m^6}$
44. $\frac{27n^9}{8}$
45. $\frac{x^{14}}{y^{11}}$
46. $\frac{b^{10}}{a^{10}}$
47. $\frac{p^{15}}{q^5}$
48. $\frac{1}{m^{10}n}$
49. 584,000,000
50. .078
51. 1.596
52. .000168
53. 260
54. .000027
55. 850,000
56. .0005